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# Force-Centric Perspectives on Autonomous Safety Maneuvers ⋆

**Lars Nielsen** *

* *Division of Vehicular Systems, Department of Electrical Engineering,
Linköping University, Sweden (e-mail: lars.nielsen@liu.se).*

**Abstract:** Real-time avoidance maneuvers have been developed using a force-centric perspective, where the founding principles are obtained from studies of optimal maneuvers. The developed optimization framework, the different criteria used, and the obtained solutions give insight into how to control the forces on the vehicle. A highlight in this presentation is the first algorithm not needing a tire-road friction estimate.

## 1. INTRODUCTION

Fully autonomous vehicles must be able to handle sudden critical situations, that may be caused by a sudden change in tire-road friction or by a suddenly appearing obstacle, like a moose on the road. In such situations it is important to be able to handle unknown or only partially known tire-road friction. Here it is interesting to compare with emergency braking using ABS, since it does not need any knowledge about friction. Instead, ABS is an algorithm using wheel speed information to secure that no wheel is locking but rolls at a slow speed seeking optimum braking. This is an example of a force-centric perspective. Effectively this means that ABS tries to optimize the braking force on the vehicle, thus minimizing the stopping distance (the actual trajectory). Note that the actual stopping distance will be different depending on the actual tire-road friction. Even so, the force-centric approach makes the best of the situation despite no knowledge about actual friction.

Consider now a sudden avoidance maneuver. A possible approach could be to use a model (including friction $\mu$) to compute a path/trajectory, and then try to follow it. Another approach, in the force-centric spirit, would be to search for an algorithm not needing friction knowledge. This presentation gives first results in this direction. The methods are inspired by ABS in the sense that they rather than trying to follow a path/trajectory strive for optimizing a certain force.

The presentation is organized so that it starts with some additional discussion on emergency braking and avoidance. Then the different steps leading up to the control principles are presented. These include an optimization framework, the formulation of the optimization problem, and analysis of the solutions. The optimization framework also proves to be beneficial to use for a new type of analysis of crash databases, providing insight into potential benefits of autonomous safety maneuvers. Finally, a real-time controller is presented, and conclusions are drawn.

## 2. EMERGENCY BRAKING

A typical longitudinal force-slip diagram can be seen in Fig. 1. When emergency braking with ABS the algorithm moves around the peak, thus both avoiding locking and trying to optimize braking force (Kiencke and Nielsen, 2005). The resulting paths/trajectories for two cases of tire-road friction are shown in Fig. 2. The figure shows two cars traveling at 70 km/h. The red line for lower friction illustrates when actual friction allows braking at $0.8g$ and the blue line is for higher friction allowing braking at $1.0g$. The force arrows are plotted for every 0.4 s. Naturally, the path is longer for lower $\mu$, but in both cases the stopping distance is close to optimal thanks to ABS.

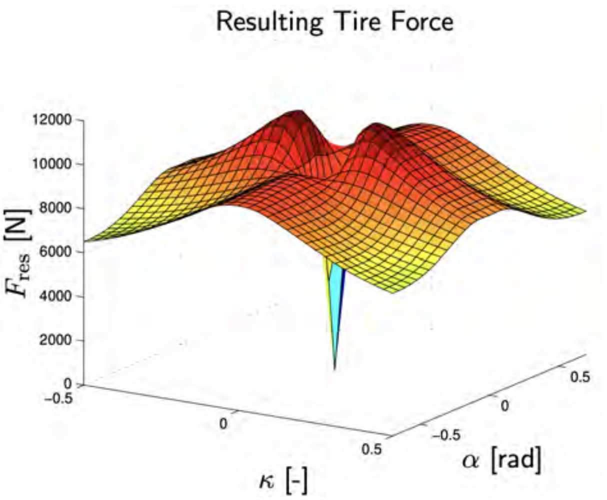

Fig. 1. Tire Model with empirical Pacejka parameters and weighting functions. For straight braking $\alpha = 0$.

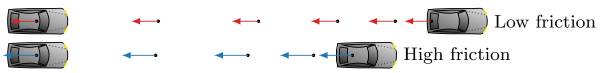

Fig. 2. Force-centric braking. ABS maximizes braking force. The resulting trajectories differ due to available tire-road friction (leading to different stopping distances).

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## 3. CRITICAL SAFETY MANEUVERS

Another principal safety maneuver is avoidance, e.g. of an obstacle on road like in Fig. 3.

Fig. 3. A fully autonomous vehicle needs to be able to handle sudden critical situations.

A first step in the research has been trying to understand optimal maneuvers. It is clear that professional race drivers and rally drivers have more ways to handle a car compared to current safety systems, Fig. 4. If these maneuvers can be understood by optimal solutions, the question then becomes if these solutions can be transferred into useful control schemes (Gao and Gordon, 2019; Zhao et al., 2021; Bobier and Gerdes, 2013; Goel et al., 2020).

Regarding the possibility to understand optimal maneuvers, the development over the past decade of readily available software for dynamic optimal control has shown substantial progress. Ten years ago a single maneuver, like a hair-pin turn could take hours to compute; now it is minutes, or even seconds depending on problem. We have used different combinations but typically the optimization problems are declared using the nlpsol interface in the framework CasADi, and subsequently solved by IPOPT, together with the MA57 linear solver. The first description of the methods and modeling needed is presented in (Berntorp et al., 2014), and it is further developed in the other references in this presentation.

Fig. 4. The capabilities of race and rally drivers are inspiration for studies in optimal dynamic control.

## 4. OPTIMIZATION FORMULATION

Compared to classical safety systems, like ESC, there are now more information available in vehicles from sensors and other sources like maps. A conseqence is that, with some certainty, there is situation awareness in the vehicle regarding lane limits, surrounding traffic, and obstacles on road, which is the basis for systems like Lane Departure Warning or Lane Keeping Control. This type of knowledge about undesired areas for the car is straightforward to include in an optimization formulation using inequalities on vehicle position to define geometric limits for its motion, like in (8). The main parts of the optimization formulation are

- Criterion $J$ in (1). To provoke at-the-limit maneuvers, minimum time or maximum speed can be used.
- Inputs are individual wheel torques, $T_i$, and steering angle, $\delta$, both with limits on value range and derivatives, see (2)-(4).
- Road and obstacle constraints $f$, in (8).
- Vehicle dynamics $G$ and $h$ in DAEs (Differential-Algebraic Equations) standard formulation in (9).

In addition there are initial conditions and final conditions. The formulation is

$$\begin{aligned}
&\text{minimize} && J && (1)\\
&\text{subject to} && T_{i,\min} \le T_i \le T_{i,\max}, && (2)\\
& && |\dot{T}_i| \le \dot{T}_{i,\max},\ i \in \{1,2,3,4\}, && (3)\\
& && |\delta| \le \delta_{\max},\ |\dot{\delta}| \le \dot{\delta}_{\max}, && (4)\\
& && X(0) = X_0,\ Y(0) = Y_0,\ \psi(0) = 0, && (5)\\
& && v_X(0) = v_0,\ v_Y(0) = 0, && (6)\\
& && X(t_f) = X_{t_f},\ Y(t_f) \le Y_{t_f}, && (7)\\
& && f(X(t), Y(t)) \le 0 && (8)\\
& && \dot{\mathbf{x}} = G(\mathbf{x}, \mathbf{z}, \mathbf{u}),\ h(\mathbf{x}, \mathbf{z}, \mathbf{u}) = 0, && (9)
\end{aligned}$$

As mentioned, this was first presented in (Berntorp et al., 2014). The modifications in modeling to handle gravel, ice, or snow are discussed in (Olofsson et al., 2013).

## 5. DIFFERENT CRITERIA

Different choices for the optimization criterion $J$ in (1) have been used. A natural choice to force an at-the-limit-of-friction maneuver is to do a maneuver as fast as possible, i.e. minimum-time control. It turns out that there are other useful choices for $J$ like maximum entry speed, $v_0$, or maximum exit speed, $v_f$, which are studied in (Fors et al., 2019). Fig. 5 shows the paths for these two cases, and Fig. 6 shows the corresponding velocities. Letting the optimization find the maximum entry speed in a curve is a straightforward way to find the maximum speed a vehicle

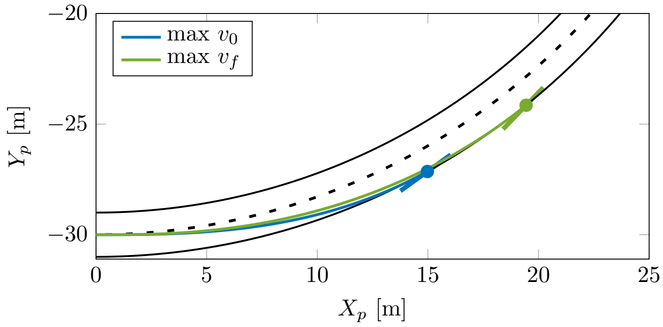

Fig. 5. Paths for maximum entry speed, $v_0$, or maximum exit speed, $v_f$. Vehicle entry from left.

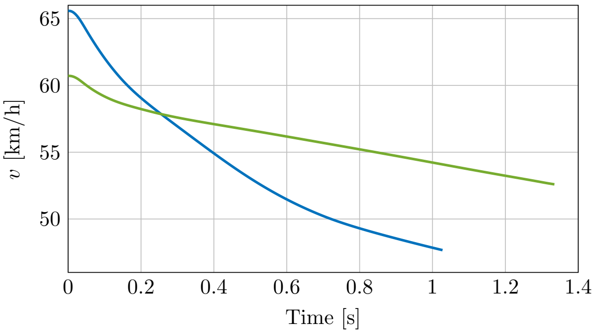

Fig. 6. Velocity profiles for maximum entry speed, $v_0$ (blue curve), and maximum exit speed, $v_f$ (green curve) for the paths in Fig. 5.

can enter a certain curve. This comes handy, e.g. when analyzing if an road-departure accident could have been avoided or not by autonomous control.

From Fig. 6 it is clear that there is much more braking for maximum entry speed since the velocity drops much faster than for maximum exit speed, $v_f$, in a curve. Looking further into this led to an interesting discovery. Fig. 7 shows the torques on the inner wheels and on the outer wheels. As can be seen in the figure there is no braking at all on the outer wheels. This means that it resembles optimal yaw control, and can therefore be used as a model for traditional ESC, which is utilized in (Olofsson and Nielsen, 2021). The paper (Fors et al., 2019) also includes the continuous family of criteria $J = \eta v_0 + (1-\eta)v_f$ where $\eta$ is an interpolation parameter.

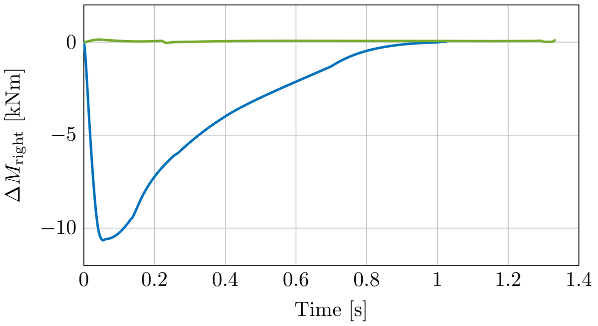

Fig. 7. For the maneuver of maximum exit speed, $v_f$, in a curve, the sum of the inner wheel torques (blue curve) and the sum of the outer wheel torques (green curve) are plotted.

The optimization framework has been used for other criteria $J$. One example is LDP, Lane Deviation Penalty, that penalizes being in the opposing lane instead of minimizing time of the whole maneuver. This in fact results in less time in the opposing lane (Anistratov et al., 2021b). The possibility of segmenting the optimization in parallel segments has been investigated (Anistratov et al., 2020).

## 6. CRASH DATABASE ANALYSIS

Before continuing with the line of thought leading to real-time control, another important use of the optimization framework will be pointed out, which is its use in crash database analysis (Olofsson and Nielsen, 2021).

There are large databases, used by e.g. insurance companies, containing more or less detailed information about accidents, and one example is GIDAS (German In-Depth Accident Study) with more than 50000 accidents stored. Data like initial speed and curve radius are included, as are the type of accident, e.g. lane departure accidents as in Fig. 8. Simulation has been the traditional tool when analyzing such data, but with the progress of optimization new questions can be efficiently handled as presented in (Olofsson and Nielsen, 2021). In that paper, for a given time frame, all 233 fatal or severe road-departure accidents from GIDAS were filtered out. They are plotted in Fig. 9. The curve in Fig. 9 is the maximum possible entry speed for the curve radius at hand, i.e. max $v_0$ for the given $R$. The paper introduces the concept of manageable

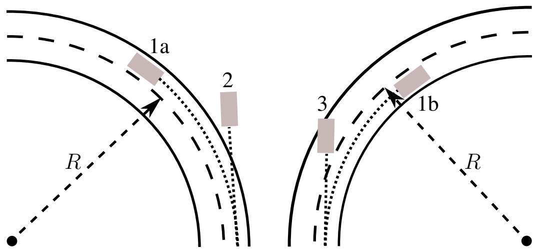

Fig. 8. Possible situations when entering a left-hand or right-hand turn with radius $R$. Cases 2 and 3 can potentially result in accidents, whereas in Cases 1a and 1b the vehicle is able to stay in the desired driving lane.

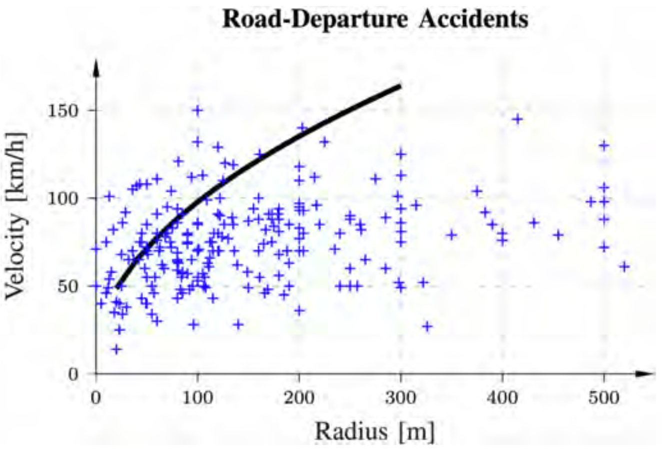

Fig. 9. All 233 road-departure accidents in GIDAS with fatal or severe outcome.

accidents which are those under the curve. (Using max $v_f$ (modeling ESC) and traditional steering with $\delta = v/R$, also these types of vehicle control are included in the paper.) The analysis in the paper gives that 197 of the 233 accidents with fatal or severe outcome could have been avoided by autonomous control (Olofsson and Nielsen, 2021). This indicates great potential in such methods, and is of course very encouraging for the development of autonomous safety maneuvers.

## 7. ATTAINABLE FORCE VOLUMES

Now, we return back to the force-centric perspective and to the principles used in real-time control. Consider the avoidance scenario in Fig. 10, which is studied in (Fors et al., 2020). That paper shows that it is useful to define a new coordinate system $F_c$ as depicted in the figure. The $x$-axis of that coordinate system is aligned with the tangent of the path at the final point where the obstacle is avoided. The $y$-axis is orthogonal to the path at the final point. This means that the $y$-axis, $F_{c,y}$, is orthogonal to the car velocity at the end, but before that it points more or less obliquely backwards relative to the car.

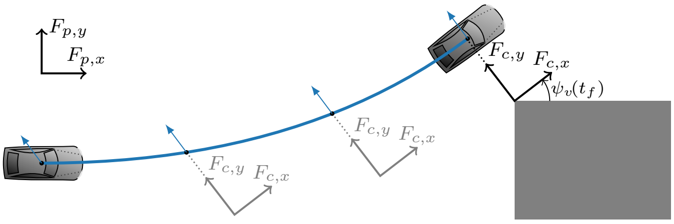

Fig. 10. Avoidance scenario with the control coordinate system $F_c$ that is fixed to ground and not to vehicle.

For this set-up, the optimal solutions for maximum entry speed in a curve, max $v_0$, were studied. For these solutions, the total forces on the vehicle were computed. They are the total forces expressed in the control coordinate system $F_{c,x}, F_{c,y}$ and the total moment $\Delta M$. For this study the concept of attainable forces was used to capture the concept of attainable actions (Yang et al., 2014), e.g. if already at the limit then the force can not be increased along certain directions. For a given time instant, and corresponding state of the vehicle, one may plot e.g. $F_{c,y}$ as function of $\Delta M$ giving an area of attainable actions. Plotting such areas of attainable forces as function of time gives an attainable force volume (Fors et al., 2020).

Fig. 11 shows the attainable force volume for the situation in Fig. 10. In the same figure, the optimal solution for max $v_0$ is plotted as a solid line. Further, the maximum of $F_{c,y}$ is indicated as a dashed line. From that visualization it is clear that the optimal solution, except for an initial transient, resides on the boundary of the volume. In fact, the optimal maneuver is close to the dashed line on top of the volume implying almost maximization of the force $F_{c,y}$. See also projection on the $F_{c,y}$-plane.

## 8. IMPORTANT CONTROL PRINCIPLE

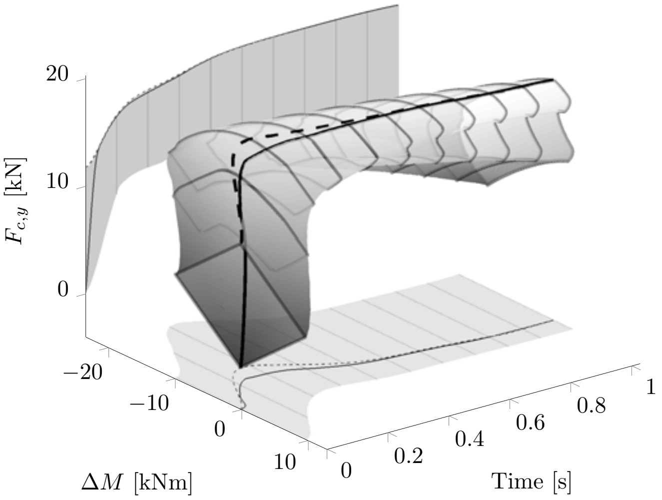

Fig. 11. Force volume for the scenario in Fig. 10. The optimal maneuver (solid line) is close to optimal $F_{c,y}$ (dashed line).

Fig. 11 gives an important control principle. Recall that the plot shows the total forces on the vehicle, $F_{c,y}$ as function of $\Delta M$, not the forces on the individual wheels resulting from individual wheel torques and steering angle. The conclusion from Fig. 11 is to maximize $F_{c,y}$ and let the total moment $\Delta M$ take the resulting values from that. This is different than traditional yaw control where emphasis is put on $\Delta M$ to control the orientation of the car. Here, instead the control principle is to control the individual wheel torques and the steering angle, so that the sum results in a maximization of $F_{c,y}$.

This control principle fits well with the knowledge from optimal control of a point mass. That problem can be solved analytically, and the solution is to have a constant force vector along a fix direction in space. This is analogous to thinking in the control coordinate system $F_c$.

## 9. REAL-TIME FORCE-CENTRIC CONTROL

The previous section established control of total forces on the vehicle as an important principle. Thus, it is now natural to look into the perspective of force-centric control. The main idea is given in Fig. 12, Fig. 13, and Fig. 14. An optimal solution to a maneuver contains all variables including paths, trajectories, and resulting forces as indicated in Fig. 12.

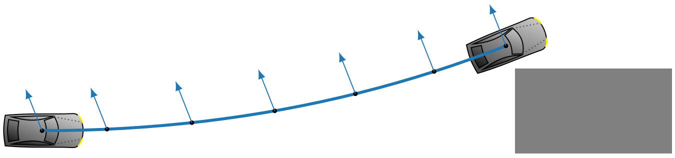

Fig. 12. An optimization computes the optimal path, trajectory, and forces.

The traditional way of using an optimal solution is to give the path or the trajectory to a basic controller that performs path and/or trajectory tracking. This idea is illustrated in Fig. 13.

The force-centric way is to instead use the forces as the basis for control, as indicated in Fig. 14. Note that it is

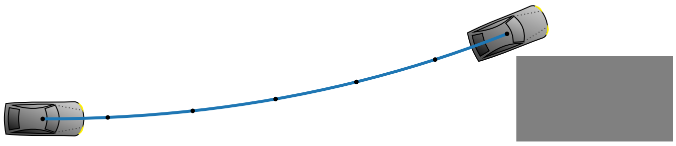

Fig. 13. A common way to use an optimal solution is to use the path or trajectory and have a controller tracking them.

the total forces on the car, and not the input torques that are used. Instead the control inputs, wheel torques and steering angle, are used to create a total force and moment on the car as in the optimal solution. A huge advantage with this approach turns out to be the same as for ABS. We can design controllers that independent of knowledge of actual friction create as large as possible force vector in the desired direction, and by that making the best of the situation. Another benefit is its real-time applicability.

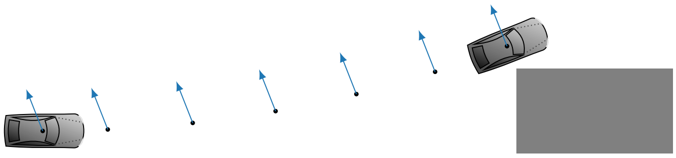

Fig. 14. The force-centric approach, advocated here, is to instead use the forces as references for the controller.

Before presenting an actual design in the next section, it is in place to give a remark about the recovery after an avoidance maneuver. Once the obstacle or situation is cleared, then the vehicle should go back to normal driving. Using optimization one should be aware that there are singularities. The simplest example is when it is optimal to drive at constant speed. Then the total force on the vehicle is constant, but since only the sum of forces is determined then there are infinitely many combinations of wheel forces giving the same total force. These aspects are investigated in (Anistratov et al., 2021a) where some methods to deal with these issues are devised.

## 10. WARY CONTROL

The control principle from Section 8 can now be realized using the force-centric perspective from Section 9. The main paper introducing and making use of these ideas is (Fors et al., 2021b), where the principal algorithm is called Wary Control. The controller strives to maximize avoidance forces, but is wary in the sense that it like ABS avoids skidding. The paper considers the avoidance situation in Fig. 15. Sensors give the distance $A$ and the angle $\gamma$. From that $\theta$ is calculated to determine the reference direction $F_{c,y}$. (The algorithm uses acceleration vector references instead of force references, but they are interchangeable.) In fact, only $\gamma$ is needed to compute $\theta$, which means that the sensor problem is fairly easy to solve. Then $\theta$ is used to give a reference for the acceleration vector of the vehicle. Force/acceleration references are generated step-wise from overall total force references on the vehicle down to force/acceleration references for the individual wheels, which in turn is used to derive the actual control signals in terms of individual wheel torques and steering angle.

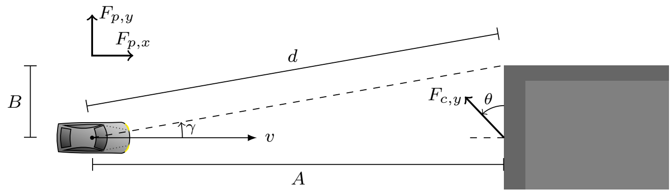

Fig. 15. An avoidance scenario.

Most of the steps are explicit vector calculations, but one particular step is to compute wheel slip given the vector-acceleration reference for the wheel. For that, a new analytical friction-limit tire model is derived in (Fors et al., 2021b). It is based on the observation that the friction-limit is elliptical and can be analytically inverted. Therefore, the whole algorithm is explicit, making the computation time negligible. In a current implementation it is around 20 assignment statements, and the whole control algorithm runs at 1000 Hz.

The next question is about performance. As shown in (Fors et al., 2021b) wary control performs close to the optimal numerical solution shown in Fig. 11. Another way of presenting this is to plot the actual force utilization for each tire, and compare it with the optimal numerical solution. This is done in Fig. 16, and it is clear that wary control is very close to the optimal solution.

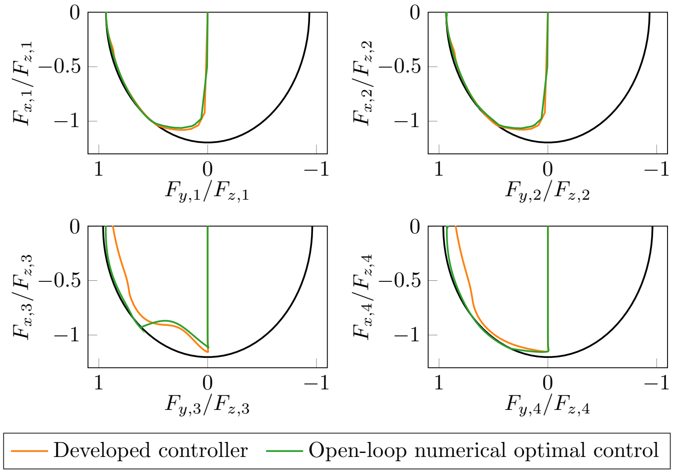

Fig. 16. Force utilization for each of the tires during the avoidance maneuver.

The controller performs well when simulated in CarMaker as reported in (Fors et al., 2021b), and also in first real tests at Stanford.

To summarize, Wary Control is characterized by

- Force/acceleration references are generated step-wise from overall total force references on the vehicle down to force/acceleration references for the individual wheels, which in turn is used to derive the actual control signals in terms of individual wheel torques and steering angle.
- The computations are explicit, and the controller has negligible computation time.
- It performs close to the optimal numerical solution.
- It performs well when simulated in CarMaker, and in first practical tests.

Finally, we connect back to emergency braking. Wary Control strives to maximize avoidance forces. This means that the magnitude of forces is different depending on what the actual friction happens to be. In turn, this means that the resulting path/trajectory becomes different depending on actual available friction. Fig. 17 shows the performance for two such cases in the same spirit as was done in Fig. 2 for ABS.

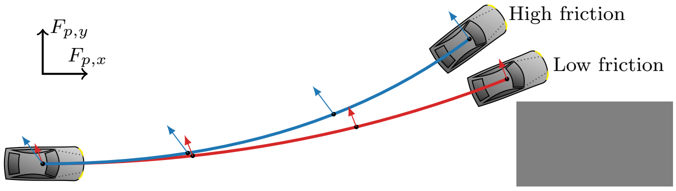

Fig. 17. Force-centric avoidance. Wary control maximizes force in a given direction. The resulting trajectories differ depending on actual friction. Compare Fig. 2.

## 11. CONSEQUENCES ON ARCHITECTURE

Any autonomous car will have a control system with several levels. For longitudinal motion it is clear that the basic level will contain ABS, i.e. force-centric control for critical situations. This paper indicates that the same may be the case for avoidance maneuvers. Even though the paradigm of path/trajectory planning together with path/trajectory tracking is well suited for higher levels of usual driving, all results presented here advocate a force-centric approach for critical safety maneuvers.

One may combine path planning and force-centric control, which is done in (Fors et al., 2021a). The planner provides reference acceleration-vectors to an acceleration vector follower (running at 1000 Hz).

## 12. CONCLUSIONS

Inspired by the success of ABS in emergency braking, a force-centric perspective has been used to design real-time avoidance control. The principles are well founded from studies of optimal control. Main advantages of the method are that, like for ABS, no knowledge about actual friction is needed, and that the computations are explicit so the computational time is negligible. So far only two papers have been published based on these ideas, but many more are foreseen since there are several design choices and possibilities within the force-centric perspective. There are also several possibilities to combine it with other methods in an architecture for autonomous driving.

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